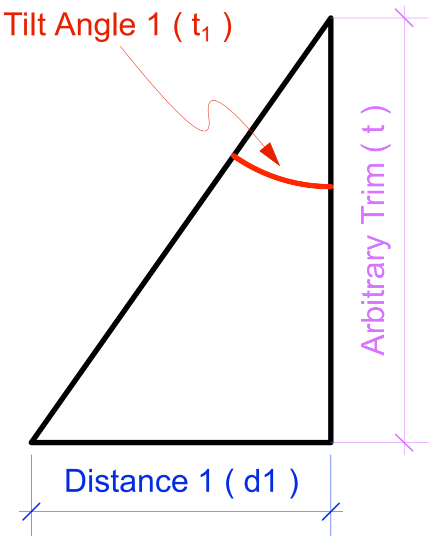
Generating Straight Line Console Curves For Moving Head Fixtures

AKA Sean Beach Struggles to Remember High School Trigonometry

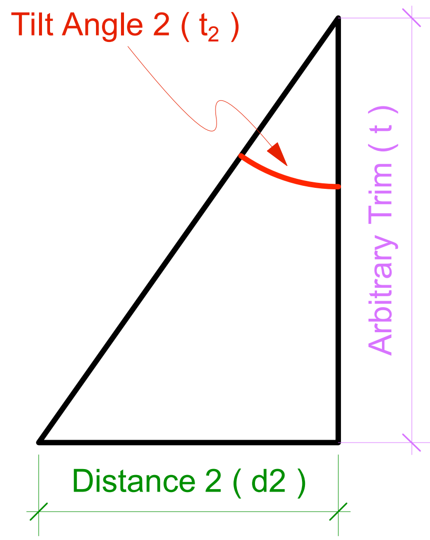
1. Determine the linear distance along the floor from the fixture to each point.
Using the tilt angle from each of our two preset positions and the fixture's trim height, we can use simple trig to find the distance. The trim height can be arbitrary for the sake of generating a console curve, provided our ground plane (stage floor) is stationary and perpendicular to our light. Looking at the light in section view, parallel to the the pan angle we get a right triangle:



SOH CAH TOA: tan(A) = o / a

$d_1 = \tan(t_1) * t$

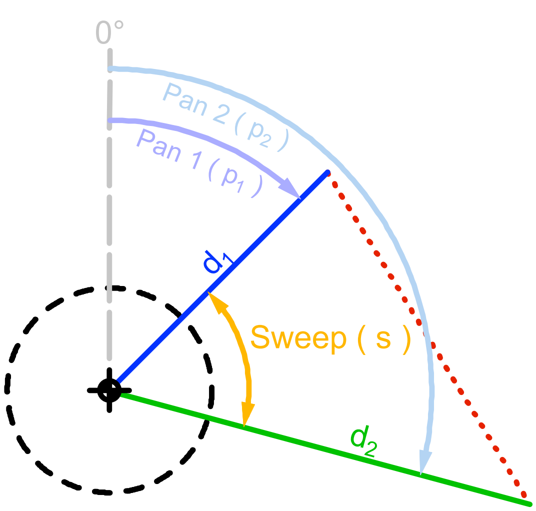
Use the tilt angle from your first preset positon to find your first distance and the TOA portion of SOH•CAH•TOA



SOH CAH TOA: tan(A) = o / a

$d_2 = \tan(t_2) * t$

Use the tilt angle from your second preset positon to find your first distance and the TOA portion of SOH•CAH•TOA



$s = \text{abs}(p_2 - p_1)$

2. Calculate your sweep angle
Determining the sweep angle is as simple as subtracting the pan value of your first preset position from the pan value of your second preset position. The absolute value of this number is your sweep angle.

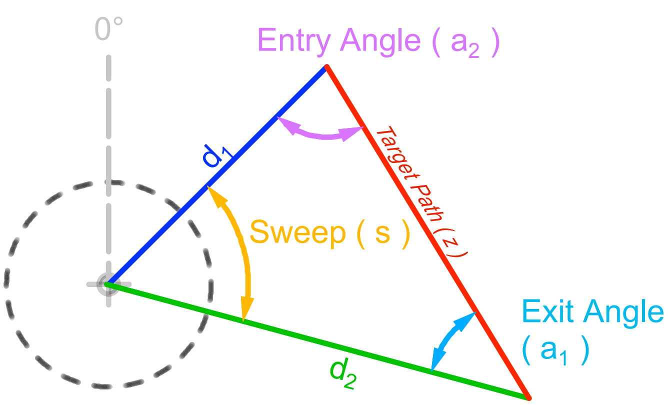
In the illustration to the left, d_1 is the distance along the floor to the first preset position, d_2 is the distance along the floor to the second preset position. The dotted red line between them is the ideal straight line path between the two.

As the sweep angle exceeds 90° the curve would actually dip to a value below either of the two tilt angles, and in some consoles would require and intermediate preset position. If the sweep angle exceeds 180° triangle-based math will no longer work ;)

3. Solve For All Other Angles and Sides

All of the formulas we use rely on having 3 pieces of information: either two sides and an angle or two angles and a side. To determine tilt values along our sweep, we'll need to know the distance to that point in the straight line. In this step we determine all the angles and sides of our plan-view shape in order to use them in later formulas.

In the illustration below d_1 and d_2 still represent the distance to our preset positions; s is our sweep angle, z is the distance between the two positions; e_1 and e_2 represent the two other angles of our triangle, though their names are not particularly helpful to understanding their purpose.



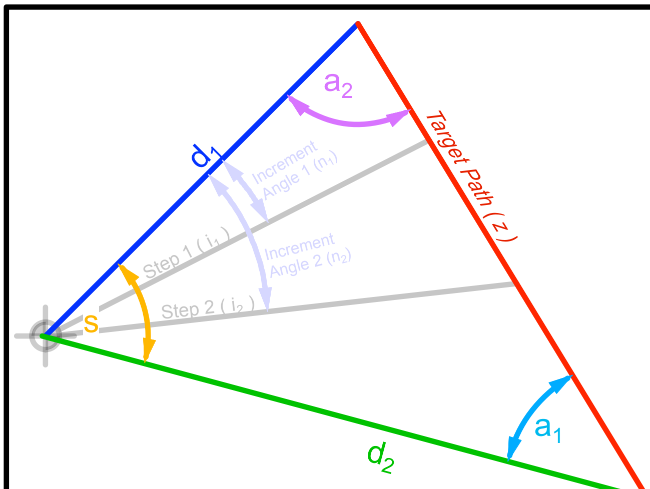
Law of Cosines: $a^2 = b^2 + c^2 - 2bc*\cos(A)$
Law of Sines: $\sin(A)/a = \sin(B)/b = \sin(C)/c$

$$z = \text{sqrt}(d_1^2 + d_2^2 - (2 * d_1 * d_2 * \cos(s)))$$

```
if(s >= 90° || d2 < d1 ) {        ' sin( e1 ) / d2 = sin( s ) / z
    a2 = sin-1( ( sin( s ) * d2 ) / z )
    a1 = 180 - a2 - s
} else {                          ' sin( e2 ) / d1 = sin( s ) / z
    a1 = sin-1( (sin( s ) * d1 ) / z )
    a2 = 180 - a1 - s
}
```

First, using the law of cosines we solve for z, the path the fixture should take. We'll use this value to calculate our other two angles, a_1 and a_2.

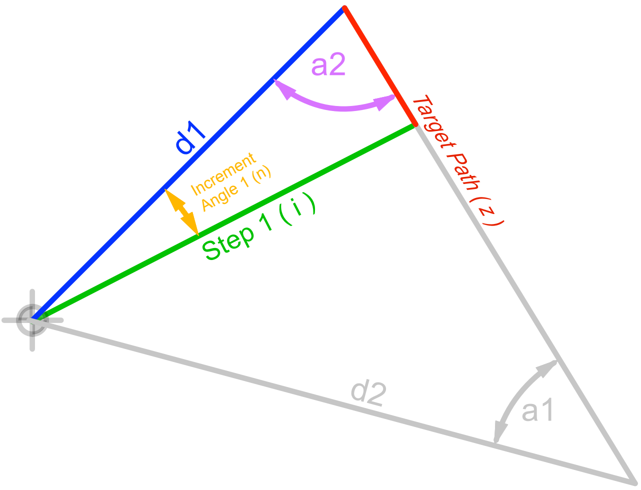
Using the law of sines we solve for one of our missing angles. We have to do this conditionally for different sweeps, if the sweep is greater than or equal to 90°, we can solve for either point. If the sweep is less than 90°, the potential exists for one of the other angles (a_1 or a_2) to be 90° or greater. Because inverse sin only returns values less than 90°, we must solve for the smaller of the two angles, the one across from the shortest side. The conditional statement above solves this problem. At this point, all of our variables are populated with values.



4. Slice It Into Pieces...

Now we know the shape of our triangle we just have to slice it up into lots of smaller triangles. The example to the right shows a curve with 4 steps, we have the first and last tilt value, so we need the two in between. Using the distance we calculated in step one (d_1), the amount we've already panned as one angle (n_1), and the angle we found in step 3 (a_2) we now have enough information to determine the distance to our intermediate point (i_1).

This step can be an iterative process that happens as many times as steps exist in your console curve utility or be fed live in a console with the current amount of pan minus the initial amount.



5. Find The Slice Distance

Working one sliver at a time we determine the distance to that point on the target path. We first find the missing angle (x) by subtracting the others from 180 degrees (the total in a triangle):

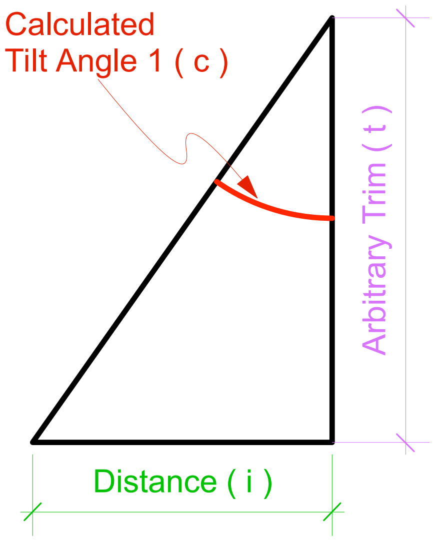
$x = 180 - a_2 - n$

Then using the law of sines we find the distance:

Law of Sines: a/Sin(A) = b/Sin(B) = c/Sin(C)

$i = (d1 * \sin(a_2)) / \sin(x)$

Now we just have everything we need to find the tilt.



6. Calculate the Tilt

Armed with the the distance to a given, calculated point along the path, finding the tilt value is as simple as reversing what we did in step one.

The tilt is calculated by finding inverse tangent of our recently calculated distance divided by our same arbitrary trim from Step 1.

SOH CAH TOA: tan(A) = o / a

$c = \tan^{-1}(i / t)$